Discussion 7 Worksheet

Date: 9/15/2021

MATH 53 Multivariable Calculus

1 Chain Rule

(a) Compute $\frac{d}{dt}f(x(t), y(t))$ for $f(x, y) = xy^3 - x^2y$, $x(t) = t^2 + 1$, $y(t) = t^2 - 1$. You don't need to express the final result in terms of t.

(b) Compute $\frac{d}{dt}f(x(t), y(t))$ for $f(x, y) = \sin x \cos y$, $x(t) = \sqrt{t}$, $y(t) = 1/t$. Express the final result in terms of t.

(c) Compute the partial derivatives of f with respect to x, y for $f(s, t) = s^2 - t^2, s = \dfrac{x+y}{2}, t = \dfrac{x-y}{2}$ using both the chain rule and by plugging in s and t.

(d) Compute the partial derivatives of $f(x, y) = \left(\sqrt{x^2 + y^2} - \frac{1}{\sqrt{x^2+y^2}}\right) e^{\frac{x^2+y^2}{2}}$ by identifying an appropriate expression $r(x, y)$ such that $f(r)$ is a simpler expression. You can assume $(x, y) \neq (0, 0)$. It's fine to have r in the final expression.

(e) Suppose z is given by $z = e^{s+2t}$, where $s = x/y$ and $t = y/x$. Compute $\partial z/\partial x$ and $\partial z/\partial y$. Express the result in x, y and z.

2 Challenge

Consider the function

$$f(x, y) = \begin{cases} \frac{x^3y - xy^3}{x^2+y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0). \end{cases}$$

Show that $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$ but exist, but are unequal. Warning: Since the second partial derivatives are discontinuous at $(0, 0)$, you will not get the right answer by evaluating $f_{xy}(a, b)$ and $f_{yx}(a, b)$ and taking the limit for $(a, b) \to (0, 0)$.

3 True/False

(a) T F Consider $f(x, y) = (\cos x + (1 + x) \tan y)e^{x^2-1+y}$. Then $\frac{d}{dt}f(t^2, t^3) = 0$ at $t = 0$.

Note: These problems are taken from the worksheets for Math 53 in the Spring of 2021 with Prof. Stankova.